Course Syllabus

Course Description

This is a Ph.D. level course in stochastic models designed to develop a solid understanding of uncertain phenomena and mathematical tools used to model and analyze random observations in industrial engineering. The course will provide both rigorous proof-based mathematical basis and related applications.

Office Hours

(a) **Yunan Liu** (*Instructor*)
446 Daniels, Thursday 3:00 - 5:00 pm
Email: *yunan_liu@ncsu.edu*
Website: *http://yunanliu.wordpress.ncsu.edu*

(b) **Kyle Hovey** (*Teaching Assistants*)
444 Daniels, Monday 2:00 - 4:00 pm
Email: *kahovey@ncsu.edu*

Course Website

http://moodle.wolfware.ncsu.edu

Prerequisites

Knowledge on probability theory and stochastic models, such as ISE589.

Reference Texts

(α) *Required:*
Ross, S. M. *Intro to Probability Models.* 10th or 11th Edition, Academic Press, Elsevier.

(β) *Recommended:*
Ross, S. M. *Stochastic Processes.* 2nd Edition, Academic Press, Weley, 1996.

Homework

There will be weekly assignments due every Tuesday in class. Graded assignments will be returned in class.

- Students are encouraged to collaborate with other students in the class, as long as each person writes his/her own solutions.
- But any such collaboration should be clearly **noted** (If some ideas of your solutions come from the discussion with another person, write his/her name on your solution).
- Copying homework from another student (past or present) is **forbidden**.
- Late homework will **NOT** be accepted.

Recitations

479 Daniels, Monday 2:00-3:00 (as part of the TA office hours). The TA helps solve selected homework problems.

Exams

All exams are in class, closed book, closed note. You are allowed to bring a two-sided cheat sheet.

- 1st midterm: September 29 (Thursday), 9:00 - 11:30.
- 2nd midterm: November 1 (Tuesday), 9:00 - 11:30.
- Final: December 8 (Tuesday), 8:00 - 11:00.

Project

There will be a project (done by a group of at most two students) which consists of two parts:

(i) Modeling real systems: Apply mathematical methods to model a real system that you encounter in your daily life (e.g., bank, highway, gym, etc.) Explain why your model is appropriate; propose methods to help improve the operational efficiency of this system; and conduct some analysis (numerical or analytic).

(ii) Popularizing OR methods and results: Choose ≥ 5 results in your OR courses (e.g., 760, 505, 723, etc.) and explain them in plain and comprehensible words. The goal is to make non-OR people (such as your dad, assuming he is not a math professor!) understand them. This will help improve your teaching skills. Albert Einstein said: "If you cannot explain it simply, you do not understand it well enough!"

The project will be due by the end of the term.

Grading

Define the following random variables:
$HW \equiv$ homework, $F \equiv$ project, $M_1 \equiv$ midterm 1, $M_2 \equiv$ midterm 2, $F \equiv$ final exam and $G \equiv$ overall grade. Then the overall grade is given by

$$G \equiv HW \times 15\% + P \times 10\% + M_1 \times 25\% + M_2 \times 25\% + F \times 30\% - \min(M_1, M_2, F) \times 5\%.$$

Tentative Course Outline

1. Review of Probability Theory
   - Probability space
   - Independence and dependence
   - Conditional probability and Bayes' formula
   - Random variables: definition, distribution functions, discrete and continuous types
   - Random variables: expectation, variance, covariance and moment generating functions
   - Markov's inequality and Chebyshev's inequality

- Modes of convergence
- Limit theorems: strong law of large number (SLLN) and central limit theorem (CLT)

2. Discrete-Time Markov Chain (DTMC)

- Definition: the Markov property
- Classification of states: transience and recurrence
- Chapman-Kolmogorov equations
- The Gambler's ruin problem
- Steady-state distributions
- DTMCs with absorbing states/classes: canonical forms, fundamental matrices, and mean times until absorption
- Time reversibility, random walk on a graph

3. Poisson Process (PP)

- Exponential distribution: the lack-of-memory property and its applications
- Equivalency of the three definitions of Poisson processes
- Properties of Poisson: independent thinning and superposition
- Order statistics and conditional distributions of the arrival times
- Generalization 1: compound Poisson process (thinning and superposition for NPP)
- Generalization 2: nonhomogeneous Poisson process (definitions, properties and connection to PP)
- The $M_t/G/\infty$ queue: number of customers at time t and the departure process

4. Continuous-Time Markov Chain (CTMC)

- CTMC: basic definition, transition probability and rate matrices
- Kolmogorov-Chapman equation and Kolmogorov ODE
- Steady state: two different approaches
- Birth-and-death processes and Markovian queueing networks
- Time reversibility

5. Renewal Counting Process (RCP)

- Renewal functions and renewal equations
- Renewal reward processes (RRP)
- Limit theorems for RCP and RRP
- Age, excess and spread of an RCP
- An application: patterns