Course Syllabus

Course Description

This course is an introduction to stochastic models with applications in industrial engineering. This course also can serve as part of an introduction to probability theory and its applications, being a sequel to ST 371 - Introduction to Probability Theory. In a first course on probability (such as ST 371) students learn about random variables and their probability distributions. In that first course, attention is usually focused on only one or two random variables. In this subsequent course we will extend the focus to stochastic processes, which are collections of random variables, usually indexed by time (which can either discrete or continuous).

Stochastic modeling appears to be an important and fundamental tool in operations research. For example, we might use stochastic processes to model the evolution of a stock price over time, the damage claims received by an insurance company over time, the work-in-process inventory in a factory over time or the number of calls waiting in telephone call center over time, all of which evolve with considerable uncertainty.

Office Hours

- **Yunan Liu** (*Instructor*)
  446 Daniels, Tuesday/Thursday 4:00 - 5:00 pm
  Email: *yunan_liu@ncsu.edu*
  Website: *http://yunanliu.wordpress.ncsu.edu/*

- **Yayun Jin** (*Teaching Assistants*)
  Office: 418 Daniels
  Recitation: 479 Daniels, Monday 2:00 - 4:00 pm
  Email: *yjin15@ncsu : edu*

Course Website

http://moodle.wolfware.ncsu.edu

Prerequisites

Basic knowledge on probability theory, such as ST 371.

Reference Texts

- (Required:) Ross, S. M. *Introduction to Probability Models.* 11th (10th) Edition, Academic Press, Elsevier.
- (Recommended:) Harchol-Balter, M. *Performance Modeling and Design of Computer Systems: Queueing Theory in Action* Cambridge University Press, 2013.

Homework

There will be weekly assignments due every Tuesday in class. Students are encouraged to collaborate with other students in the class, as long as each person writes his/her own solutions. Copying homework from another student (past or present) is forbidden. Graded assignments will be returned in class.

Recitations

479 Daniels, Monday 2:00-4:00 (as part of the TA office hours). The TA helps solve selected homework problems.

Exams

All exams are in class, open textbook, open notes.

- 1st midterm: October 6 (Tuesday), 1:15 - 2:50.
- 2nd midterm: November 3 (Tuesday), 1:15 - 2:50.
- Final: December 8 (Tuesday), 1:00 - 4:00.

Project

There will be a project (by a group of two students). Apply mathematical methods to model a real system that you encounter in your daily life (e.g., bank, highway, gym, etc.) Explain why your model is appropriate; propose methods to help improve the optional efficiency of this system; and conduct some analysis and experiments (numerical or analytic).

Grading

Define the following random variables:
$HW \equiv$ homework, $P \equiv$ project, $M_1 \equiv$ midterm 1, $M_2 \equiv$ midterm 2, $F \equiv$ final exam and $G \equiv$ overall grade. Then the overall grade is given by

$$G \equiv HW \times 15\% + P \times 10\% + M_1 \times 25\% + M_2 \times 25\% + F \times 30\% - \min(M_1, M_2, F) \times 5\%.$$

Tentative Course Outline

1. Review on Probability Theory
   - Probability space
   - Independence and dependence
   - Conditional probability and Bayes' formula
   - Random variables: definition, distribution functions, discrete and continuous types
   - Random variables: expectation, variance, covariance and moment generating functions
   - Limit theorems: strong law of large number (SLLN) and central limit theorem (CLT)
2. Discrete-Time Markov Chain (DTMC)
   - Definition: the Markov property
   - Classification of states: transience and recurrence
   - Chapman-Kolmogorov equations

- The Gambler's ruin problem
- Steady-state distributions
- DTMCs with absorbing states/classes: canonical forms, fundamental matrices, and mean times until absorption
- Time reversibility, random walk on a graph

3. Poisson Process (PP)
    - Exponential distribution: the lack-of-memory property and its applications
    - Definitions of Poisson processes
    - Properties of Poisson: independent thinning and superposition
    - Order statistics and conditional distributions of the arrival times
    - Generalization 1: compound Poisson process
    - Generalization 2: nonhomogeneous Poisson process
4. Continuous-Time Markov Chain (CTMC)
    - CTMC: basic definition, transition probability and rate matrices
    - Kolmogorov-Chapman equation and Kolmogorov ODE
    - Steady state: two different approaches
    - Birth-and-death processes and Markovian queueing networks
    - Time reversibility