Course Syllabus

Course Description

This is a course on *stochastic modeling with an emphasis on queueing theory*, as a natural continuation of the ISE Ph.D. qualifier course ISE 760. One goal is to help students learn about various application context. A second goal is to focus on a class of mathematical models and analysis techniques that have proven useful in the application context. As is almost always the case in operations research, these models and analysis techniques have many other applications, so that the course can be useful even if you are primarily interested in other applications.

From the mathematical perspective, the course consists of both the conventional single-server queues and the recently developed multi-server queues (and networks of such multi-server queues). Important customer behavior includes *abandoning* (leaving after waiting for a while), retrying (coming back later after abandoning) and returning (coming back for additional service). There may be multiple types of customers and customer service representatives (agents) with different sets of skills. We use matrix analytic methods to study Markovian queueing systems. The course also covers non-Markovian queues.

Time and Place

Tuesday and Thursday 1:30–2:45. Room: Daniels 216.

Instructor

Yunan Liu
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Teaching Assistant

Korhan Aras
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Textbooks

(i) S. Ross, *Introduction to Probability Models*. 11th Edition, Academic Press, 2014.

(ii) S. Ross, *Stochastic Process*. 2nd Edition, Academic Press, Weley, 1996.

Recommended books

(i) G. Latouche and V. Ramaswami, *Introduction to Matrix Analytic Methods in Stochastic Modeling*. Society for Industrial and Applied Mathematics, 1987.

(ii) S. Karlin and H. Taylor, *A First Course in Stochastic Processes*. Academic Press, 1975.

(iii) S. Asmussen, *Applied Probability and Queues*. Springer, 2003.

Prerequisites

This course is intended for graduate students in operations research and related fields. Student are expected to have completed a first course on stochastic models at the level of the first-year doctoral course ISE 760.

Homework

There will be five to six homework assignments. Students are encouraged to collaborate with other students in the class, as long as each person writes his/her own solutions. Copying homework from another student (past or present) is forbidden. Graded assignments will be returned in class.

Exams

All exams are open notes. You are allowed to bring a two-sided cheat sheet.

- Midterm: March 17.
- Final: TBD.

Grading

Define the following random variables:
$HW \equiv$ homework, $M \equiv$ midterm, $F \equiv$ final exam and $G \equiv$ overall grade.
Then the overall grade is given by

$$G \equiv HW \times 20\% + M \times 40\% + F \times 40\%.$$

Tentative Course Outline

The course topics include:

1. Renewal processes (continued) and semi-Markov processes
   - CLT for renewal processes
   - Stopping times and Ward's equation
   - Renewal equation and its numerical solutions
   - Key renewal theorem
   - Equilibrium renewal processes
   - Semi-Markov chains
2. Brownian motion (BM)
   - From random walks to Brownian motions
   - Gaussian processes
   - Hitting times and maximum values of BM

- Variations of BM:
  (i) BM absorbed at a value; (ii) Reflected BM; (iii) Brownian bridge; (iv) Geometric BM; (v) Integrated BM; (vi) Fractional BM; (vii) multi-dimensional BM; (viii) Brownian sheet; (ix) BM with drift.
- Option pricing and the Black-Scholes formula
- Stochastic differential equations (SDEs) and Itô's formula
- Ornstein-Uhlenbeck (OU) processes

3. Phase-type (PH) distributions
   - Special cases of PH distributions
   - Continuous PH distributions
   - Discrete PH distributions
   - Closure properties
   - Queueing models with PH distributions
4. Markovian queueing models
   - Open Jackson networks
   - Closed Jackson networks
   - Semi-open Jackson networks
5. Non-Markovian queueing models
   - Pollaczek-Khintchine formula and $M/GI/1$ queues
   - Lindley recursion and $GI/GI/1$ queues
   - Other queueing models with non-exponential distributions.
6. Other key queueing results
   - Little's law (LL)
   - Rate conservation law (RCL)
   - Poisson arrival sees time average (PASTA)