Introduction to Queueing Theory and Applications

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Motivation

What in common?

Motivation

People wait in a line!

Foundation of Operations Research

A Little History

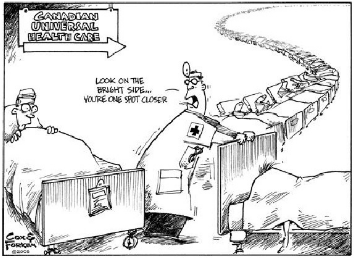
- Father of queueing theory: Anger Erlang (1878–1929)
- Originally used to model telephone exchange

Queues Are Everywhere!

- Real queues
- Virtual queues
- Systems transformed into queues.

Applications: Real Queues

Hospitals



Applications: Real Queues

Transportation

Applications: Real Queues

Airport Security Lines

Applications: Real Queues

DMV

Applications: Real Queues

Bank

Applications: Real Queues

Amusement Parks

Applications: Virtual Queues

Contact Centers

Applications: Virtual Queues

Computer Service Systems

Applications: Virtual Queues

Housing

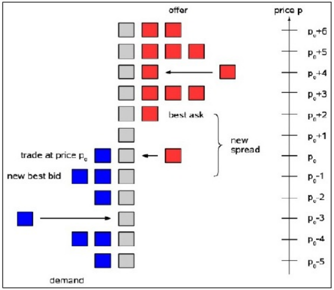
Applications: Transformed Queues

Manufacturing Systems

Applications: Transformed Queues

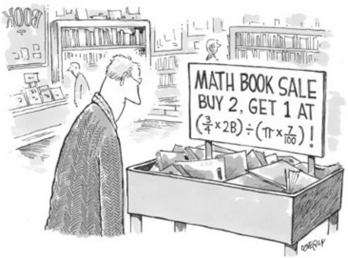
Inventory Systems

High Frequency Trading: Order Books



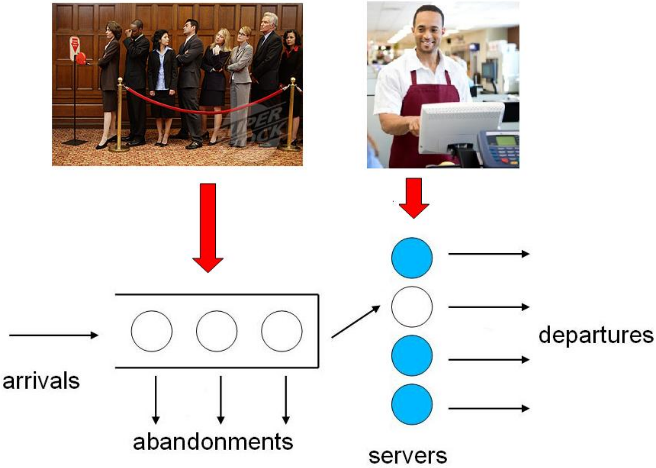
Queues Are Indeed Everywhere!

Mathematical Queueing Models

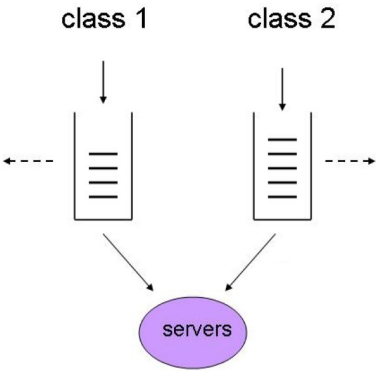


Queueing Models

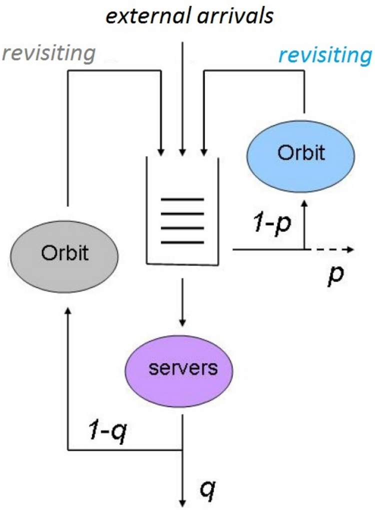
Single-class queues



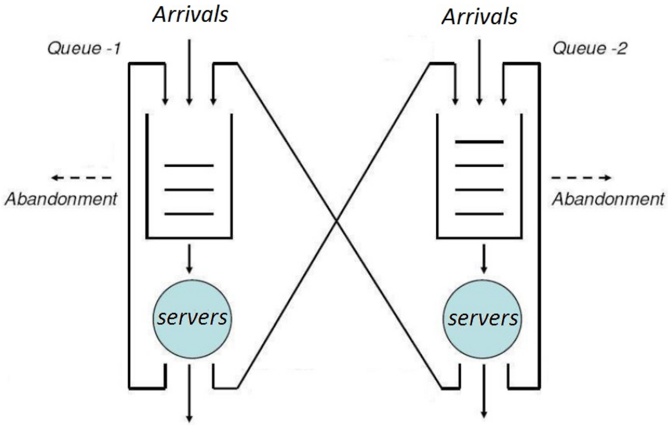
Multi-class queues



Re-entrant queues



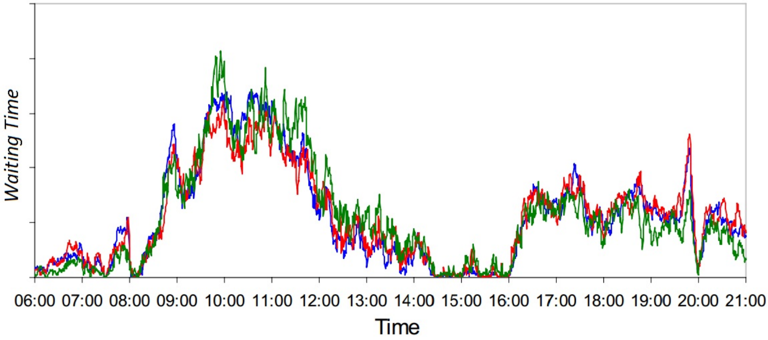
Queueing Networks



Realistic Real-World Features

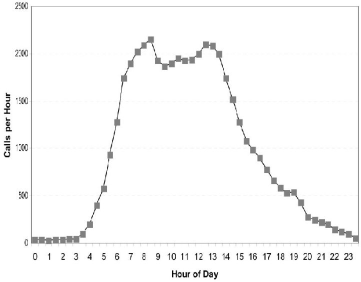
Realistic Models Features

Random variables and processes

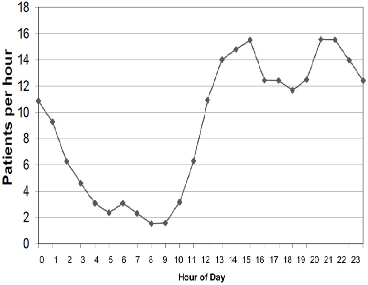


Realistic Models Features

Time-varying arrivals



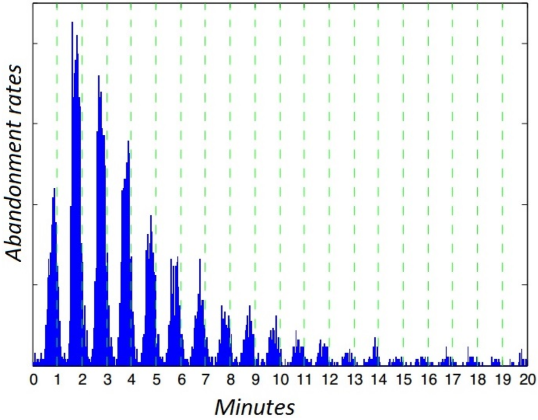
call center



emergence room

Realistic Models Features

Strange customer behavior



Realistic Models Features

Different service disciplines/policies

- first-come first-served (FCFS):
- last-come first-served (LCFS):
- processor sharing (PS):
- Shortest job first (SJF):
- Priority preemptive (PP):

Different service disciplines/policies

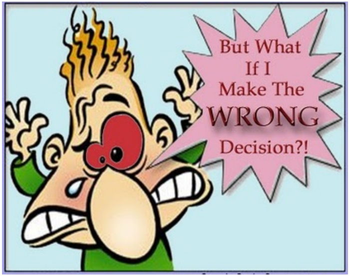
- first-come first-served (FCFS):
  used in most service systems
- last-come first-served (LCFS):
  computer stack operations, inventory systems with perishable products
- processor sharing (PS):
  computer systems
- Shortest job first (SJF):
  computer systems, CPU scheduling
- Priority preemptive (PP):
  emergency rooms, service systems with multiple classes

Analyze The Models and Obtain Quantitative Results

- How? .... (hereby omit 10,000 pages)
- The "......" part will be taught in our undergraduate/graduate courses

Help Make Decisions

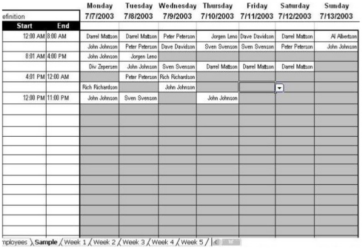
- Relieve human suffering of waiting
- Minimize costs/maximize profits
- Save lives!
- What else?



Staffing to Achieve System Stability

Design staffing and shifts

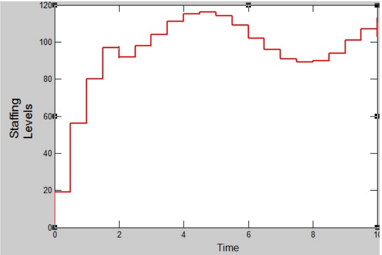
Shift schedule



efinition		Monday 7/7/2003	Tuesday 7/8/2003	Wednesday 7/9/2003	Thursday 7/10/2003	Friday 7/11/2003	Saturday 7/12/2003	Sunday 7/13/2003
Start	End							
12:00 AM	8:00 AM	Darrel Mattson	Darrel Mattson	Peter Peterson	Jorgen Leno	Dave Davidson	Darrel Mattson	Al Albertson
		John Johnson	Peter Peterson	Dave Davidson	Sven Svenson	Sven Svenson	Peter Peterson	John Johnson
8:01 AM	4:00 PM	John Johnson	Jorgen Leno					
		Div Zepersen	John Johnson	Sven Svenson	Darrel Mattson	Darrel Mattson	Darrel Mattson	
4:01 PM	12:00 AM		Peter Peterson	Rich Richardson				
		Rich Richardson		John Johnson				
12:00 PM	11:00 PM	John Johnson	Sven Svenson		John Johnson			

mployees / Sample / Week 1 / Week 2 / Week 3 / Week 4 / Week 5

Number of servers

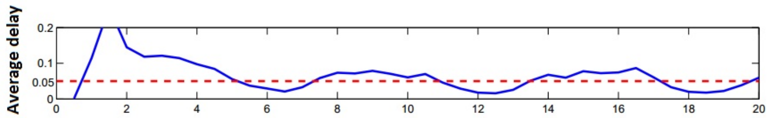


Meet service level agreements

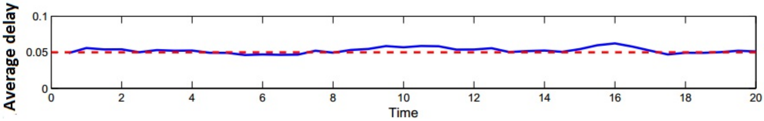
- $P(\text{waiting} < 2 \text{ mins}) > 0.8$
- $E(\text{wait}) \approx 3 \text{ mins} = 0.05 \text{ hr}$
- $P(\text{Abandonment}) < 0.02$

Staffing to Achieve System Stability

Bad staffing



Good staffing



Delay Forecasting and Announcement

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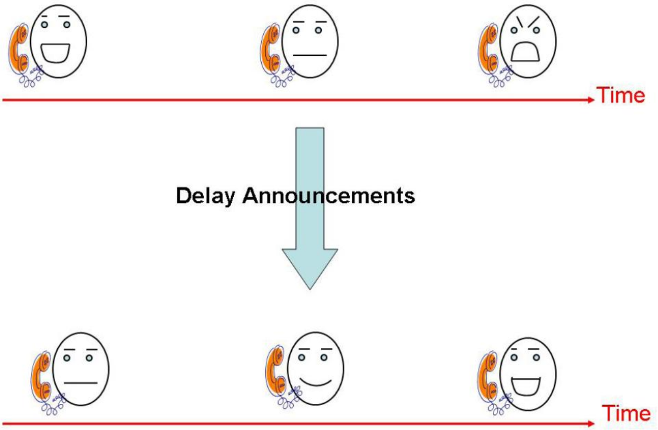
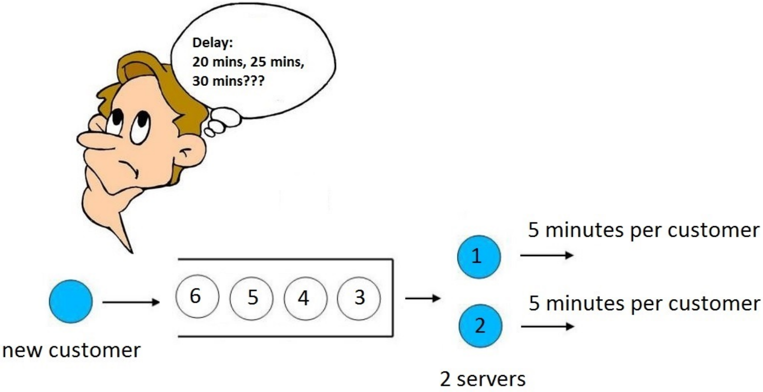


Delay Forecasting and Announcement: An Exercise



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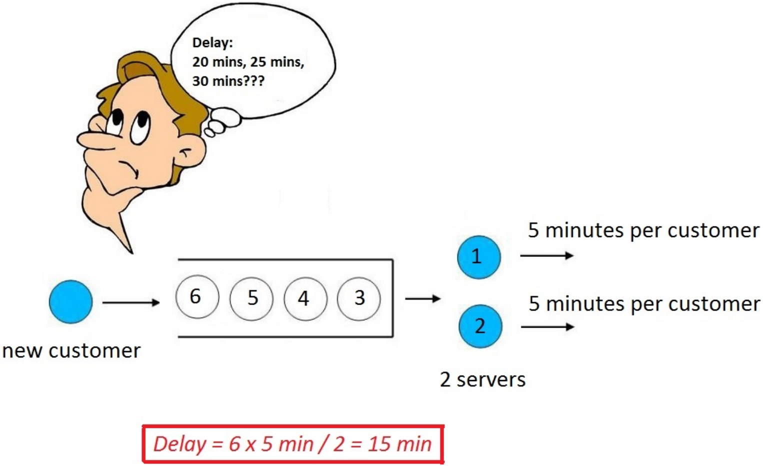
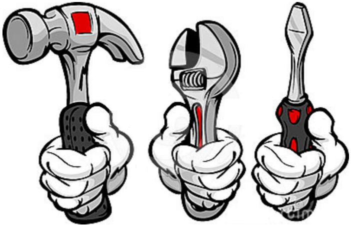


Tools

- Data analysis: analyze data, test hypothesis, abstract information, etc.
- Computer simulation: discussed earlier today
- Probability theory: model and predict random events



Probability Exercise 1: Patterns

The pattern $HTHH$ occurs at step 11, 14 and 22 in the sequence:

$$HHTHTTT\underline{HTH\underline{H}}\underline{THH}HTTT\underline{HTHH}TTH\ldots$$

Now consider 4 patterns:

$$\mathcal{A} \equiv HH, \quad \mathcal{B} \equiv HT, \quad \mathcal{C} \equiv TT \quad and \quad \mathcal{D} \equiv TH.$$

Q: Which one "on average" appears in the smallest number of flips?

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- $4 = N_{\mathcal{B}} < N_{\mathcal{A}} = 6$.

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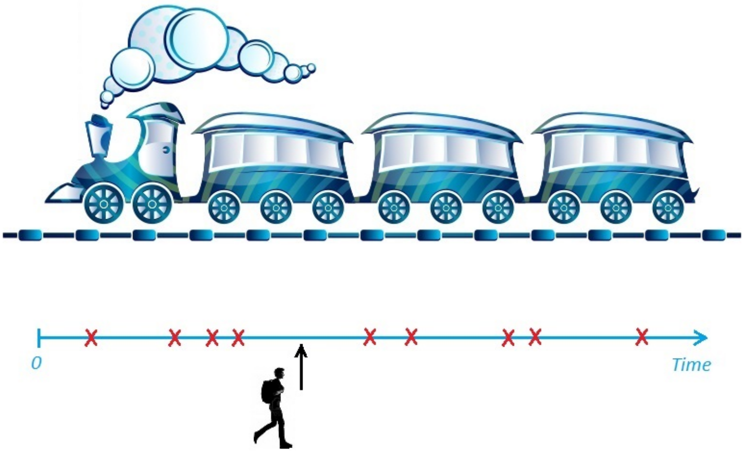
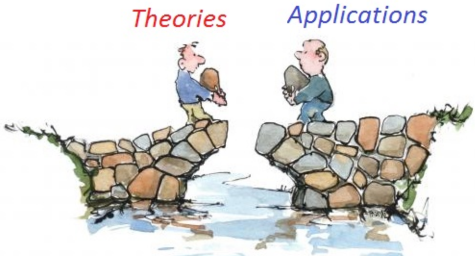
Probability Exercise 2: NYC Subway Problem



Conclusion

- Observe real-world systems and recognize potential problems
- Construct mathematical models representing these systems
- Analyze the models (performance analysis and decision making)
- Use the analysis to provide strategies, heuristics and insights
- Solve real-world problems (connect theories and applications)



Thank You!