ISE Summer Camp Queueing Theory

Yunan Liu

Introduction to Queueing Theory and Applications

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ISE Summer Camp, June 24, 2013

Motivation history Applications Queueing Models Realistic Features Decision Making Useful Tools

Motivation

Bankof America



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What in common?





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Motivation

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Motivation

People wait in a line!







Foundation of Operations Research



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A Little History



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- ► Father of queueing theory: Anger Erlang (1878–1929)
- Originally used to model telephone exchange

Queues Are Everywhere!

- Real queues
- Virtual queues
- Systems transformed into queues.

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Hospitals





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Transportation



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Airport Security Lines



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Amusement Parks





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Applications: Virtual Queues Contact Centers







lour call is important to us, just not as important as whatever else we're doing.

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Applications: Virtual Queues

Computer Service Systems



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Applications: Virtual Queues

Housing



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Applications: Transformed Queues

Manufacturing Systems





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Applications: Transformed Queues

Inventory Systems



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Applications: Transformed Queues

High Frequency Trading: Order Books





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Queues Are Indeed Everywhere!

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Mathematical Queueing Models



Single-class queues



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Multi-class queues



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Re-entrant queues



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Queueing Networks



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Realistic Models Features

Random variables and processes



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Realistic Models Features

Time-varying arrivals



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Strange customer behavior



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Realistic Models Features

Different service disciplines/policies

- first-come first-served (FCFS):
- Iast-come first-served (LCFS):

- processor sharing (PS):
- Shortest job first (SJF):
- Priority preemptive (PP):



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Realistic Models Features

Different service disciplines/policies

- first-come first-served (FCFS): used in most service systems
- last-come first-served (LCFS): computer stack operations, inventory systems with perishable products
- processor sharing (PS): computer systems
- Shortest job first (SJF): computer systems, CPU scheduling
- Priority preemptive (PP): emergency rooms, service systems with multiple classes

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Analyze The Models and Obtain Quantitative Results

- How? (hereby omit 10,000 pages)
- The "....." part will be taught in our undergraduate/graduate courses

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Help Make Decisions

- Relieve human suffering of waiting
- Minimize costs/maximize profits
- Save lives!
- What else?



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Staffing to Achieve System Stability

Design staffing and shifts

Shift schedule

afinition		Monday 7/7/2003	Tuesday 7/8/2003	Wednesday 7/9/2003	Thursday 7/10/2003	Friday 7/11/2003	Saturday 7/12/2003	Sunday 7/13/2003
Start	End							
12:00 AM	1:00 AM	Darel Matson	Darrel Mattoon	Peter Peterson	Jorgen Leso	Dave Davidson	Darrel Mattoos	Al Abertio
		John Johnson	Peter Peterson	Dave Davidson	Sven Svenson	Sves Svensce	Peter Peterson	John Johnso
8.01 AM	4:00 PM	John Johnson	Jorgen Lena					
		Div Zeperson	John Johnson	Sven Svenson	Darel Matson	Darrel Matson	Darrel Matson	
4.01 PM	12:00 AM		Peter Peterson	Rich Richardson				
		Rich Richardson		scandol, ndol.		2	•	
12:00 PM	11.00 PM	John Johnson	Sven Svenoce		John Johnson			
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Meet service level agreements

- P(waiting < 2 mins)>0.8
- $E(wait) \approx 3 \text{ mins} = 0.05 \text{ hr}$
- P(Abandonment)<0.02</p>

Staffing to Achieve System Stability

Average delay 0.2 0.1 0.05

Average delay 0.1 0.05 0

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Delay Forecasting and Announcement

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Delay Forecasting and Announcement



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Delay Forecasting and Announcement: An Exercise

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Delay Forecasting and Announcement: An Exercise

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Tools

- Data analysis: analyze data, test hypothesis, abstract information, etc.
- Computer simulation: discussed earlier today
- Probability theory: model and predict random events



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The pattern *HTHH* occurs at step 11, 14 and 22 in the sequence:

ННТНТТТНТН<u>НТНН</u>НТТТ<u>НТНН</u>ТТН...

Now consider 4 patterns:

 $\mathcal{A} \equiv HH$, $\mathcal{B} \equiv HT$, $\mathcal{C} \equiv TT$ and $\mathcal{D} \equiv TH$.

Q: Which one "on average" appears in the smallest number of flips?

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$$\blacktriangleright N_{\mathcal{A}} = N_{\mathcal{C}}, \ N_{\mathcal{B}} = N_{\mathcal{D}}$$

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$$\blacktriangleright N_{\mathcal{A}} = N_{\mathcal{C}}, \ N_{\mathcal{B}} = N_{\mathcal{D}}$$

• $N_{\mathcal{B}} < N_{\mathcal{A}}$? or $N_{\mathcal{B}} > N_{\mathcal{A}}$?

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The pattern HTHH occurs at step 11, 14 and 22 in the sequence:

ННТНТТТНТН<u>НТНН</u>НТТТ<u>НТНН</u>ТТН...

Now consider 4 patterns:

 $\mathcal{A} \equiv HH$, $\mathcal{B} \equiv HT$, $\mathcal{C} \equiv TT$ and $\mathcal{D} \equiv TH$.

- Q: Which one "on average" appears in the smallest number of flips?
- $N_{\mathcal{A}} = N_{\mathcal{C}}, N_{\mathcal{B}} = N_{\mathcal{D}}$ $N_{\mathcal{B}} < N_{\mathcal{A}}? \text{ or } N_{\mathcal{B}} > N_{\mathcal{A}}?$ $4 = N_{\mathcal{B}} < N_{\mathcal{A}} = 6.$

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The pattern HTHH occurs at step 11, 14 and 22 in the sequence:

HHTHTTTHTH<u>HTHH</u>HTTT<u>HTHH</u>TTH...

Now consider 4 patterns:

 $\mathcal{A} \equiv HH, \quad \mathcal{B} \equiv HT, \quad \mathcal{C} \equiv TT \quad and \quad \mathcal{D} \equiv TH.$

 Pattern HH:
 $T \dots TH$ H Done!

 Pattern HT:
 $T \dots TH$ T Done!

 Pattern HT:
 $T \dots TH$ H Done!

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Probability Exercise 2: NYC Subway Problem





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Conclusion

- Observe real-world systems and recognize potential problems
- Construct mathematical models representing these systems
- Analyze the models (performance analysis and decision making)
- Use the analysis to provide strategies, heuristics and insights
- Solve real-world problems (connect theories and applications)



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Thank You!

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